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published in

*NIC Symposium 2006* ,
G. Münster, D. Wolf, M. Kremer (Editors),
John von Neumann Institute for Computing, Jülich,
NIC Series, Vol. **32**, ISBN 3-00-017351-X, pp. 325-332, 2006.



http://www.fz-juelich.de/nic-series/volume32

# Interdisciplinary Monte Carlo Simulations

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Biological, linguistic, sociological and economical applications of statistical physics are reviewed here. They have been made on a variety of computers over a dozen years, not only at the NIC computers. A longer description can be found in[1], an emphasis on teaching in[2].

## 1 Introduction

The Monte Carlo methods invented for physics problems half a century ago were later also applied to fields outside of physics, like economy[3], biology[4], or sociology[5]. Instead of atoms one simulates animals, including people. These physics methods are often called "independent agents" when applied outside physics, to distinguish them from "representative agent" approximations and other mean field theories. "Emergence" in these fields is what physicists call self-organization, that means systems of many simple particles showing complex behaviour (like freezing or evaporating) which is not evident from the single-particle properties.

The three people cited in Refs.3-5 were not physicists; two got the economics Nobel prize. But also physicists have entered these fields intensively in the last years (and much earlier for biology; see Erwin Schrödinger's question: What is life?). The German Physical Society has since several years a working group on socio-economic problems, started by Frank Schweitzer. And our university just got approved a new Special Research Grant (SFB) where geneticists and theoretical physicists are supposed to work together. The NIC Research Group in Jülich is an earlier physics-biology example.

An important difference between physics and applications outside physics is the thermodynamic limit. A glass of Cologne beer has about $10^{25}$ water molecules, which is close enough to infinity for physicists. Economists, in contrast, are less interested in stock markets with $10^{25}$ traders. Thus finite-size effects, which often are a nuisance in Statistical Physics simulations, may be just what we need outside of physics.

Of this large area of computer simulations by physicists for fields outside physics I now select: population genetics, language competition, opinion dynamics, and market fluctuations, mostly following Ref. 1, 2.

## 2 Population Genetics

Darwinian Evolution is similar to thermal physics in that two effects compete: Mother Nature wants to select the fittest and to minimize energy; but more or less random accidents (mutations in biology, thermal noise or entropy in statistical physics) lead to deviations from ideality, like biological ageing or minimization of the free energy. The following example is ongoing work together with Cebrat, Pękalski, Moss de Oliveira and de Oliveira and can be regarded as an improved Eigen quasispecies model.

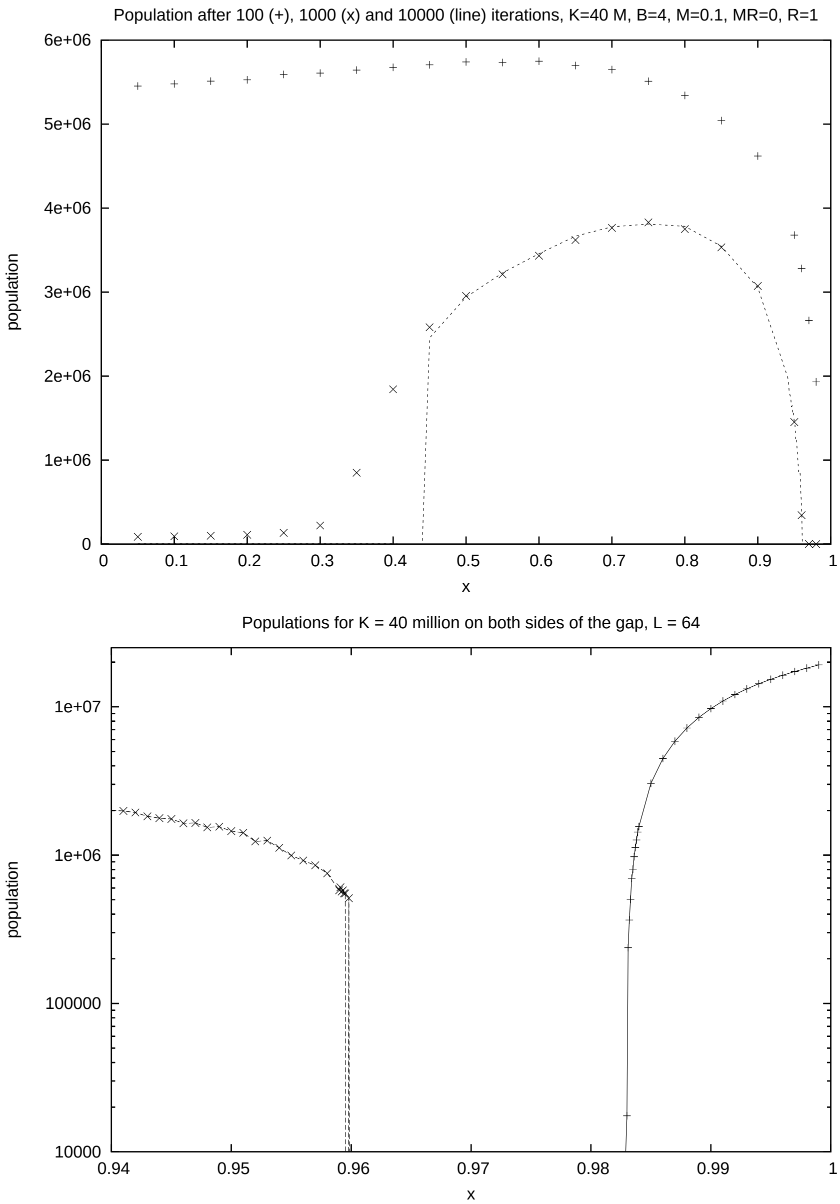


Figure 1. $M = 0.1$, $M_R = 0$, $R = 1$, $B = 4$, $L = 64$. Top part: First and second phase transition, for various observation times; the third one at $x = 0.983$ is not shown for clarity. Bottom part: Expanded semilogarithmic view of second and third phase transition.

Each individual in the population has a genome, which consists of two bit-strings inherited from the mother and the father, respectively. Each bit-string has $L$ bits with $L = 8, 16, 32, 64$, as is convenient for Fortran words (byte to integer*8). A bit set to one means

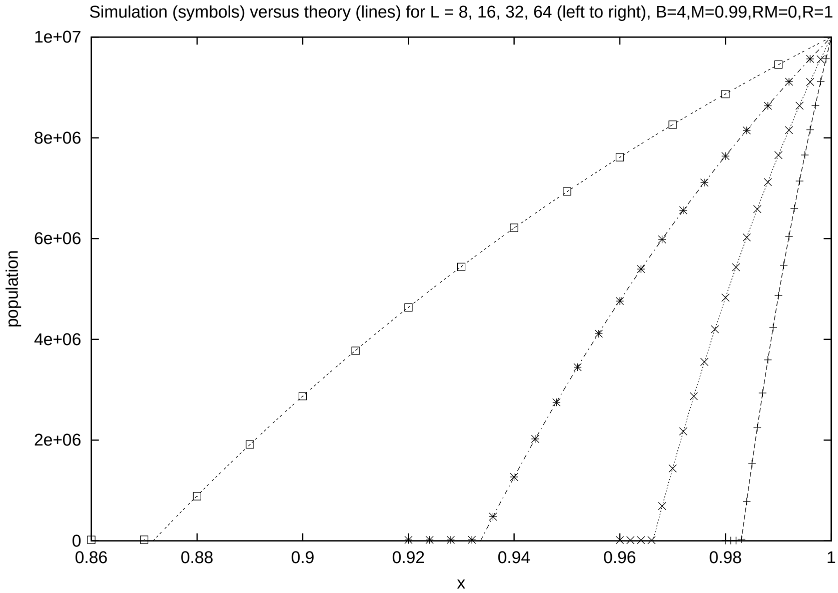


Figure 2. Simulation (symbols) versus theory (lines) for the large-$x$ region at $L = 8$, 16, 32 and 64 (from left to right).

a bad mutation in the DNA, while a zero bit is a healthy gene. All mutations are assumed to be recessive, that means they diminish the survival probability by a factor $x < 1$ if and only if both the paternal and the maternal bit-string have their corresponding bits mutated. At reproduction, the bit-strings in both the father and the mother are mutated with probability $M$ at a randomly selected position; then with probability $R$ they undergo a crossover (recombination) at some randomly selected position (like in genetic algorithms); then the bits neighbouring the crossover point are mutated with probability $M_R$; and finally one bit-string of the mother and one of the father give one child genome, with $B$ such births per iteration and per female. (The mother selects the father at random.) Mutation attempts for an already mutated bit leave this bit unchanged.

At each iteration the genetic survival probability is $x^n$ where $n$ is the number of active mutations (bit-pairs set to 1) and $x$ an input parameter. To account for limitations in space and food, as well as for infections from other individuals, additional Verhulst death probabilities proportional to the current number of individuals are applied to both the newborns and at each iteration to the adults.

For very small $x$, only mutation-free individuals survive: $n = 0$. With growing $x$ the survival chances grow, but so does the mutation load $< n >$ which in turn reduces the survival chances. As a result, for $L = 64$ three different phase transitions can be found in Fig.1: For $0 < x < 0.45$ the population dies out; for $0.45 < x < 0.96$ it survives; for $0.96 < x < 0.98$ it dies out again, and for $0.98 < x < 1$ it survives again. The transitions at 0.45 and 0.96 seem to be first-order (jump in population and load) while the one at 0.98 is second-order (continuous). For $x > 0.98$ all bits of both bit-strings are mutated to one, which allows a simple scaling prediction of the population for general $L$ in agreement with

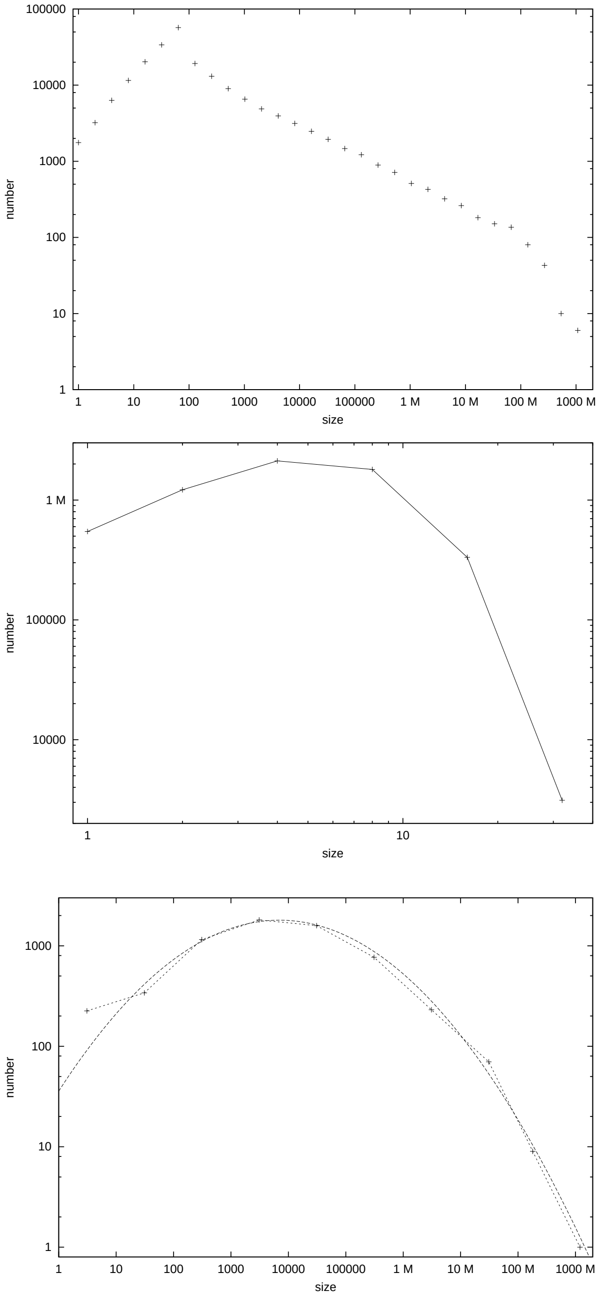


Figure 3. Distribution of language sizes in Viviane model[8] (top), in Schulze model[7] (middle) and reality[9] (bottom). The curve in the bottom part is a log-normal fit.

the simulations: Results depend on $x^L$ as seen in Fig.2. For example, the critical point at birth rate $B$ is at $x = (1 + B/2)^{-1/L}$.

Real animals get old with increasing age, and that can be simulated with similar techniques. The more complicated Penna bit-string model[6] simulates the ageing of individuals and agrees well with the empirical Gompertz law of 1825, that the mortality of adult humans increases exponentially with age[1].

## 3 Language Competition

Every ten days on average one human language dies out. Simulations of the bit-string Schulze model are very similar to the above population genetics, with random mutations, transfer of language bits from one language to another, and flight from small to large languages[7]. The alternative Viviane model[8] simplifies mutation and flight from small to large languages into one process, and ignores transfer. It gives in Fig.3 a wide range of language sizes, i.e. of the number of people speaking one language, from dying languages with only one speaker, to Chinese with $10^9$ speakers. The Schulze model gives a more realistic nearly log-normal shape for this distributions, but not the wide range of language sizes. Both the proper shape and the large size range of reality (bottom part of Fig.3) might come from non-equilibrium statistics.

In the last version of the Schulze model, each language (better interpretation: its grammar) is characterized by $F$ features each of which can adopt one of $Q$ different integer values $1, 2, ...Q$. Each site of a large square lattice is occupied by a person speaking one language. At each iteration, each feature of each person is mutated with probability $p$. This mutation is random with probability $1 - q$ while with probability $q$ the corresponding feature from one of the four lattice neighbours is adopted. Also, at each iteration, each person independently, with a probability proportional to $1 - x^2$ abandons the whole language and adopts the language of one randomly selected person in the population.

In the last version of the Viviane model, each lattice site is either empty of carries a population with a size randomly fixed between 1 and, say, like 127. Initially one lattice site is occupied and all others are empty. Then at each time step one empty neighbour of an occupied site is occupied with a probability proportional to the number of people which can live there. Then this new site adopts the language of one of its four lattice neighbours, with a probability proportional to the size of the language spoken at that neighbour site. However, this adopted language is mutated to a new language with probability inversely proportional to the new size of the adopted language. (This denominator is not allowed to exceed a maximum, set randomly between 1 and, say, 2048.) The whole process ends once the last lattice site has become occupied.

## 4 Opinion Dynamics

Can a single person make a difference in public life? In chaos theory we ask whether a single butterfly in Brazil can influence a hurrican in the Caribbean. Kauffman[4] asked the analogous question whether a single biological mutation has a minor effect or disturbs the whole genetic network[4]. Physicists call this damage spreading and ask, for example, how the evolution of an Ising model is changed if one single spin is flipped and otherwise the system, including the random numbers to simulate it, remains unperturbed. This question was discussed[10,1] for three models: The opportunists of Krause and Hegselmann[11], the negotiators of Deffuant et al[12], and the missionaries of Sznajd[13].

The opportunists take as their new opinion the average opinion of the large population to which they belong, except that they ignore those who differ too much from their own opinion. Also the negotiators ignore opinions which differ too much from their own; otherwise a randomly selected pair gets closer in their two opinions without necessarily agreeing fully. A randomly selected pair of missionaries, neighbouring on a lattice or network, convinces its neighbours if and only if the two people in the pair have the same opinion. Simulations show that the opinion change of a single person may influence the whole population for suitable parameters[10,1].

For the missionaries on a scale-free network, simulations agreed nicely with election results in Brazil, apart from fitted scale factors, Fig.4.

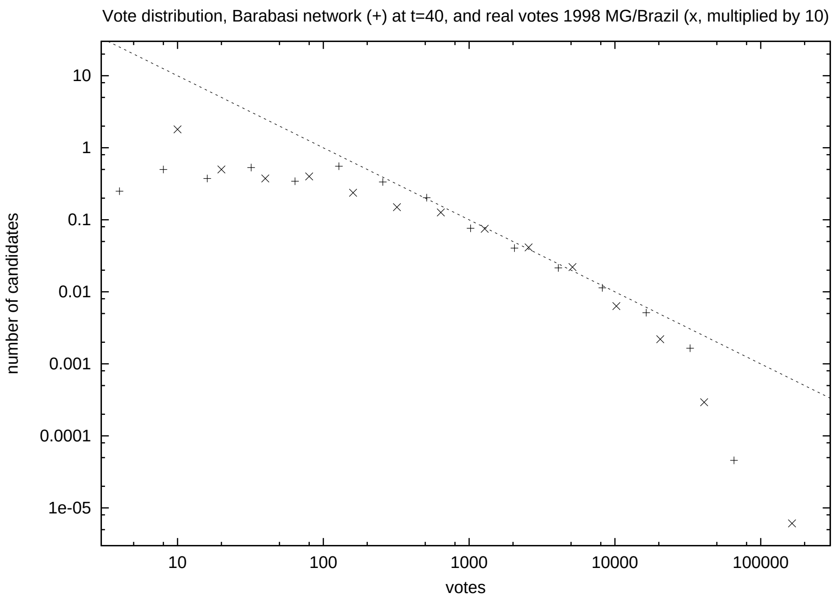


Figure 4. Distribution of the number of candidates getting a certain number of votes in simulations[14] and in elections in Minas Gerais, Brazil.

## 5 Market Fluctuations

How can we get rich fast by speculating on the stock market? This writer earned about one Heugel (a local currency unit of about $10^4$ Euro) by believing some theory for the Tokyo stock market[15]. Details, of course, are given out only for more JUMP time. Instead this section summarizes the Cont-Bouchaud model of stock market fluctuations[16], because it is closest to the pre-existing physics model of percolation.

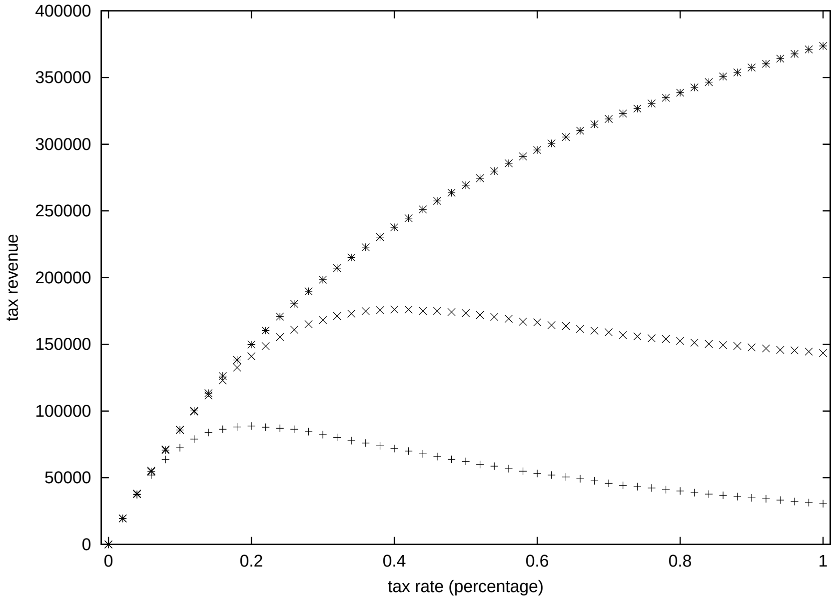


Figure 5. Tax revenue for the government versus percentage of Tobin tax to be paid for each transaction, in various versions of the Cont-Bouchaud model[17].

Each site of a large square lattice is either occupied by an investor (with probability $p$), or empty with probability $1 - p$. Sets of occupied neighbours are called clusters and are identified with groups of investors which act (buy or sell) together. At each iteration a cluster either buys (with probability $a$), sells (also with probability $a$) or sleeps (with probability $1 - 2a$). The traded amount is proportional to the number of investors in the trading cluster. The difference between supply and demand drives the market values up and down. This basic model gives on average: i) as many ups as downs on the market; ii) a power-law decay ("fat tail") for the probability to have a large price change, and with modifications also: iii) volatility clustering (markets have turbulent and calm times), iv) effective multi-fractality, v) sharp peaks and flat valleys for the prices, but no prediction on how the market will move tomorrow.

Apart from these nice basic properties also practical applications were made[17]: Does a small "Tobin" tax of a few tenths of a percent on all transactions reduce fluctuations and earn tax revenue without killing the whole market? It does, but apart from more government control over individuals there is another danger which can be simulated: If the tax revenue increases with increasing tax rate, then governments will be tempted to increase this tax again and again (as Germans just saw in fall 2005 and German students may observe in future tuition hikes.) Much better is a maximum of tax revenue at some moderate tax rate; then the government should settle on this moderate tax rate, provided it regards the simulations as reliable. Fig.5 shows that in this model such a desirable maximum exists for some parameters but not for all. Another application is the confirmation that halting the trade when excessive price changes are observed indeed helps to calm the market.

## 6 Discussion

Interdisciplinary applications of physics methods are no longer as exotic as they were years ago; biologists and economists have started to publish papers together with computational physicists on these non-physics fields.

Thanks to S. Cebrat, P.M.C. de Oliveira and S. Moss de Oliveira for comments on the manuscript.

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